

Measuring Cosmological Parameters

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Invited Review given at the 18th Texas Symposium held in Chicago, December 1996. To be published by World Scientific Press, eds. A. Olinto, J. Frieman, and D. Schramm.

February, 1997.

MEASURING COSMOLOGICAL PARAMETERS

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In this review, the status of measurements of the matter density (Ω_m), the vacuum energy density or cosmological constant (Ω_Λ), the Hubble constant (H_0), and ages of the oldest measured objects (t_0) are summarized. Measurements of the statistics of gravitational lenses and strong gravitational lensing are discussed in the context of limits on Ω_Λ . Three separate routes to the Hubble constant are considered: the measurement of time delays in multiply-imaged quasars, the Sunyaev-Zel'dovich effect in clusters, and Cepheid-based extragalactic distances. Globular-cluster ages plus a new age measurement based on radioactive dating of thorium in a metal-poor star are briefly summarized. Limits on the product of $H_0 t_0$ are also discussed. Many recent, independent dynamical measurements are yielding a low value for the matter density ($\Omega_m \sim 0.2-0.3$). A wide range of Hubble constant measurements appear to be converging in the range of 60-80 km/sec/Mpc. Areas where future improvements are likely to be made soon are highlighted; in particular, measurements of anisotropies in the cosmic microwave background. Particular attention is paid to sources of systematic error and the assumptions that underlie many of the measurement methods.

1 Introduction

Rapid progress is being made in measuring the cosmological parameters that describe the dynamical evolution and the geometry of the Universe. In essence, this is the first conclusion of this review. The second conclusion is that despite the considerable advances, the accuracy of cosmological parameters is not yet sufficiently high to discriminate amongst, or to rule out with confidence, many existing, competing, world models. We as observers still need to do better. Fortunately, there are a number of opportunities on the horizon that will allow us to do so.

In the context of the general theory of relativity, and assumptions of large-scale homogeneity and isotropy, the dynamical evolution of the Universe is specified by the Friedmann equation

$$H^2 = \frac{8\pi G \rho_m}{3} - \frac{k}{a^2} + \frac{\Lambda}{3}$$

where $a(t)$ is the scale factor, $H = \frac{\dot{a}}{a}$ is the Hubble parameter (and H_0 is the Hubble “constant” at the present epoch), ρ_m is the average mass density, k is a curvature term, and Λ is the cosmological constant, a term which represents the energy density of the vacuum. It is common practice to define the matter

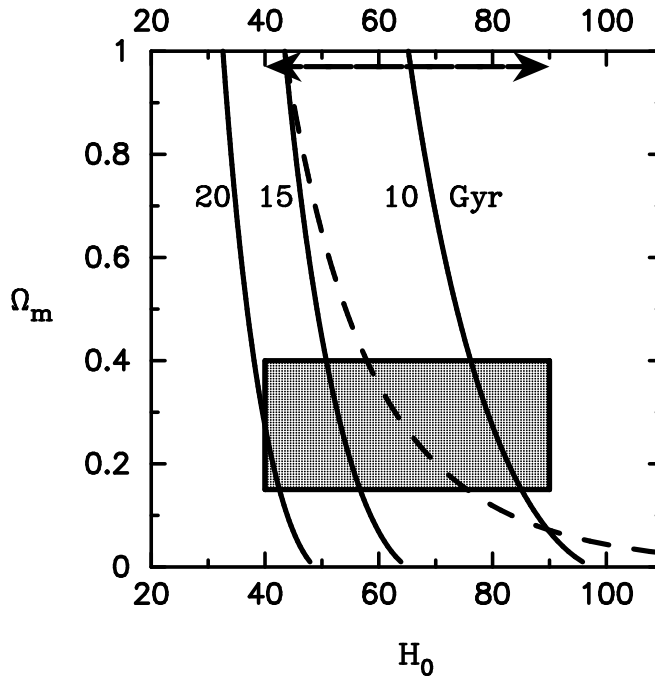


Figure 1: Ω_m versus H_0 showing current observational limits on cosmological parameters. Solid lines denote expansion ages for an open ($\Omega_\Lambda = 0$) Universe and the dashed line denotes an expansion age of 15 Gyr in the case of a flat ($\Omega_\Lambda \neq 0$) Universe. See text for details.

density ($\Omega_m = 8\pi G\rho_m/3H_0^2$), the vacuum energy density ($\Omega_\Lambda = \Lambda/3H_0^2$), and the curvature term ($\Omega_k = -k/a_0^2H_0^2$) so that $\Omega_m + \Omega_\Lambda = 1$ for the case of a flat universe where $k = 0$. The simplest case is the Einstein-de Sitter model with $\Omega_m = 1$ and $\Omega_\Lambda = 0$. The dimensionless product $H_0 t_0$ (where t_0 is the age of the Universe) is a function of both Ω_m and Ω_Λ . In the case of the Einstein-de Sitter Universe

$$f(\Omega_m, \Omega_\Lambda) = H_0 t_0 = \frac{2}{3}$$

Bounds on several cosmological parameters are summarized in Figure 1 in a plot of the matter density as a function of the Hubble constant, following Carroll, Press & Turner (1992). Solid lines represent the expansion ages for 10, 15, and 20 Gyr in an open ($\Lambda = 0$) model. The grey box is defined by values of H_0 in the range of 40 to 90 km/sec/Mpc and $0.15 < \Omega_m < 0.4$. The solid arrow

denotes the same range in H_0 for $\Omega_m = 1$. This plot illustrates the well-known “age” problem; namely that for an Einstein-de Sitter Universe ($\Omega = 1$, $\Lambda = 0$), H_0 must be less than ~ 45 km/sec/Mpc if the ages of globular clusters (t_0) are indeed ~ 15 billion years old. This discrepancy is less severe if the matter density of the Universe is less than the critical density, or if a non-zero value of the cosmological constant is allowed. For example, the dashed line indicates an expansion age of 15 Gyr in the case of a flat ($\Omega_m + \Omega_\Lambda = 1$) model for $\Lambda \neq 0$.

A number of issues that require knowledge of the cosmological parameters remain unresolved at present. First is the question of timescales ($H_0 t_0$) discussed above; possibly a related issue is the observation of red (if they are indeed old) galaxies at high redshift. Second is the amount of dark matter in the Universe. As discussed below, many dynamical estimates of the mass over a wide range of scale sizes are currently favoring values of $\Omega_m \sim 0.25 \pm 0.10$, lower than the critical Einstein-de Sitter density. And third is the origin of large-scale structure in the Universe. Accounting for the observed power spectrum of galaxy clustering has turned out to be a challenge to the best current structure formation models.

Taking the current data at face value, there appears to be a conflict with the standard Einstein-de Sitter model. In fact, it is precisely the resolution of these problems that has led to a recent resurgence of interest in a non-zero value of Λ (*e.g.* Ostriker & Steinhardt 1995; Krauss & Turner 1995). Another means of addressing these issues (*e.g.* Bartlett *et al.* 1995) requires being in conflict with essentially all of the current observational measurements of H_0 ; from purely theoretical considerations, a very low value of H_0 (≤ 30) could also resolve these issues.

Ultimately we will have to defer to measurement as the arbiter amongst the wide range of cosmological models (and their very different implications) still being discussed in the literature. A wealth of new data is becoming available and progress is being made in the measurement of all of the cosmological parameters discussed below: the matter density, Ω_m , the vacuum energy density, Ω_Λ , the expansion rate H_0 , and age of the oldest stars t_0 . The central, critical issues now are (and in fact have always been) testing for and eliminating sources of significant systematic error.

2 Ω_m – The Matter Density

Table 1 presents a summary of several different techniques for measuring the matter density of the Universe. These techniques have been developed over a wide range of scales, from galaxy (~ 100 -200 kpc), through cluster (Mpc),

on up to more global scales (redshifts of a few). Excellent, recent reviews on determinations of Ω can be found in Dekel, Burstein & White (1997) and Bahcall, Lubin & Dorman 1995 and references therein. The first part of the table lists Ω_m determinations that are independent of Ω_Λ ; the second part lists Ω_m determinations that are not independent of Ω_Λ ; and the third part of the table lists Ω_Λ determinations. In addition to listing the physical basis of the method, types of object under study, and values of Ω_m plus an estimated uncertainty, Table 1 makes explicit some of the assumptions that underlie each of these techniques. Although in many cases, 95% confidence limits are quoted, these estimates must ultimately be evaluated in the context of the validity of their underlying assumptions. It is non-trivial to assign a quantitative uncertainty in many cases, but in fact systematic effects may be the dominant source of uncertainty. Several of these assumptions and uncertainties are discussed further below. They include, for example, diverse assumptions about mass tracing light, mass-to-light ratios being constant, clusters being representative of the Universe, clumping of X-ray gas, non-evolution of type Ia supernovae, and the non-evolution of elliptical galaxies. For methods that operate over very large scales (gravitational lensing and type Ia supernovae), assumptions about Ω_Λ or Ω_{total} are currently required to place limits on Ω_m .

Since lower values of the matter density tend to be measured on smaller spatial scales, it has given rise to the suspicion that the true, global value of Ω_0 must be measured on scales beyond even those of large clusters, *i.e.*, scales of greater than ~ 100 Mpc (*e.g.*, Dekel 1994). In that way, one might reconcile the low values of Ω_m inferred locally with a spatially flat Universe. However, recent studies (Bahcall, Lubin & Dorman 1995) suggest that the M/L ratios of galaxies do not continue to grow beyond a scale size of about ~ 200 kpc (corresponding to the sizes of large halos of individual galaxies). In their Jeans analysis of the dynamics of 16 rich clusters, Carlberg *et al.* (1997) also see no further trend with scale. Hence, currently the observational evidence does not indicate that measurements of Ω_m on cluster size scales are biased to lower values than the true global value.

A brief description of several techniques for measuring the matter density is given below. These methods are discussed in the context of both their strengths and weaknesses, paying particular attention to the underlying assumptions. An excellent and more complete review on this topic is given by Dekel, Burstein & White (1997); also see Trimble (1987).

Table 1: SUMMARY OF Ω_m and Ω_Λ DETERMINATIONS

Sample	Method	Scale	Assumptions	Ω_m	Error
Ω_Λ Independent Methods					
Galaxies	dyn. M/L ratio	100 kpc	galaxies representative M/L constant	~ 0.1	
Clusters	dyn. M/L ratio	<few Mpc	clusters representative M/L constant	~ 0.2	
Clusters	X-ray M/L ratio	<few Mpc	hydrostatic eqm	~ 0.2	
Clusters	baryon fraction		clusters representative no clumping	0.3-0.5	
Clusters	morphology		model dept.	> 0.3	
Local Group	Least action principle	1 Mpc	LG representative no external torques model uniqueness	~ 0.15	
Galaxies	Virial Theorem (pairwise velocities)	1-300 Mpc	mass-indept. biasing	0.2-0.4	
Galaxies	Peculiar velocities	100 Mpc	point masses biasing	> 0.3	95%
Sample	Method	Scale	Assumptions	Ω_m	Error
Ω_Λ Dependent Methods					
Type Ia SNaE	Hubble diagram	$z < 0.5$	$\Omega_\Lambda = 0$ no evolution effects	0.88	90%
Lensed QSO's	lensing statistics	global	$\Omega_{tot}=1$ $\Omega_\Lambda=0$ dark matter distrib. slow galaxy evolution	> 0.49 > 0.15	95% 90%
6 lenses	strong lensing	global	dust small effect $\Omega_\Lambda = 0$	low Ω	
CMB	multipole analysis	global	model dependent CDM	0.3 - 1.5	
Sample	Method	Scale	Assumptions	Ω_Λ	Error
Type Ia SNaE	lensing statistics strong lensing age discrepancy	$z < 0.5$	$\Omega_{tot}=1$	< 0.51	95%
Lensed QSO's		global	$\Omega_{tot}=1$	< 0.66	95%
6 lenses		global	$\Omega_{tot}=1$	< 0.9	95%
$H_0 t_0$		100 Mpc	$H_0 > 65$ $t_0 > 13$ Gyr	> 0.5	66%

2.1 Galaxies and Clusters: Dynamical Measures & Mass-to-Light Ratios

The contribution of galaxies to the mass density can be determined by integrating the luminosity function per unit volume for galaxies and multiplying by an (assumed, constant) mean mass-to-light (M/L) ratio. The dynamical masses of galaxies can be determined from rotation curves for spiral galaxies, or the measurement of velocity dispersions and application of the virial theorem both for individual elliptical galaxies. The latter method can also be applied for groups and clusters of galaxies (as Zwicky did in the 1930's).

This method has several advantages. First it is conceptually simple and model-independent. Unlike some of the global techniques discussed below, this method is independent of both H_0 and Ω_Λ . However, there are a number of underlying assumptions. Most important is *the assumption that galaxies trace all mass*. In addition, there are implicit, underlying assumptions concerning the similarity of mass-to-light ratios in different systems (ignoring, for example, potential differences in initial mass functions, star formation histories, dark remnant populations, dust content, etc.) The estimates based on this method tend to yield low values of Ω_m of ≤ 0.25 .

2.2 Dynamics of the Local Group

Peebles (1994) estimated Ω_m by calculating the orbits of galaxies in the Local Group based on observed radial velocities, positions, and distances. Shaya *et al.* (1995) extended this method to a catalog of galaxies within 3000 km/sec. Again this is a method that is conceptually straightforward and independent of H_0 and Ω_Λ . Moreover, since the galaxies are nearby, the errors in the distances are relatively small. However, only one (the radial) component of the motion is measured. This method too is based on *the assumption that galaxies trace mass*. It also assumes that external tidal influences and past mergers are not significant. Furthermore, the question of uniqueness is difficult to address. The estimates based on this method again give low values of Ω_m of ~ 0.15 .

2.3 Cluster Baryon Fraction

This issue was discussed in detail by White *et al.* (1993) for the Coma cluster, and has been addressed now in many contexts by a number of authors (*e.g.*, White & Frenk 1991; White & Fabian 1995; Steigman & Felten 1995). The calculation goes as follows: First, the number density of baryons (Ω_b) can be determined based on the observed densities of light elements from big-bang nucleosynthesis. Hence, the fraction of baryons (f_b) measured in clusters of galaxies can be used to estimate of the overall matter density assuming

$$f_b = \frac{M_{\text{gas}}}{M_{\text{TOT}}} = \frac{M_b}{M_{\text{TOT}}} = \frac{\Omega_b}{\Omega_m}$$

There are four explicit assumptions made:

- 1) The gas is in hydrostatic equilibrium.
- 2) There is a smooth potential.
- 3) Most of the baryons in the clusters are in the X-ray gas.
- 4) The cluster baryon fraction is representative of the Universe.

If the gas is clumped or there is another source of pressure (magnetic fields or turbulence) in addition to the thermal pressure, the baryon fraction would be decreased and the matter density would be increased (Steigman & Felten 1995).

Recent measurements of X-ray clusters (*e.g.*, Loewenstein & Mushotsky 1996; White & Fabian 1995) indicate that the baryon fraction has a range of values from about 10-20%. The values for f_b tend to be smaller for small groups and in the inner regions of larger clusters. These results underscore the importance of ensuring that such measurements are made on large enough scales to be truly representative of the large-scale Universe as a whole.

Taken at face value, the cluster-baryon method estimates again favor low values of Ω_m . For $\Omega_b h^2 = 0.024 \pm 12\%$ (Tytler, this conference) relatively low values of $\Omega_m < 0.5$ are favored for the range of baryon fractions observed. The Tytler *et al.* 1997 baryon determination is at the high end of recent measures of this quantity (low end of the deuterium abundance measurements); lower baryon densities only serve to decrease the Ω_m estimates. (However, see the discussion by Bothun, Impey and McGaugh 1997; these authors suggest that perhaps low-surface-brightness galaxies could be source of most of the baryons in the Universe and that rich clusters are not representative of the overall baryon density.)

2.4 Peculiar Velocities: Density and Velocity Comparisons

On scales of ~ 100 Mpc, the motions of field galaxies can be used to infer the mass density given independent distance information. These methods do not yield a measure of Ω_m directly, but rather yield the ratio $\beta = \Omega^{0.6}/b$ where b is the bias parameter (describing the relation between mass and light) over a scale of a few hundred km/sec. These methods are again insensitive to both H_0 and Ω_Λ . Several different approaches have been investigated. For more details, the reader is referred to Dekel (1994), Willick *et al.* (1997) and Dekel, Burstein and White (1997).

All methods make use of radial velocity catalogs and distances based on the Tully-Fisher relation. The analyses differ in detail and there are advantages and disadvantages to each type of approach. At the present time, the results from this type of technique have not yet yielded a consistent picture. Earlier analyses (*e.g.* Dekel *et al.* 1993) suggested large values of $\beta \sim 1.3$, and correspondingly rather high values of Ω (subject to assumptions about the value of b). More recently, the estimates of β have decreased somewhat (Dekel, Burstein & White 1997). At present, the results from different groups (*e.g.*, Dekel, Willick, Davis and collaborators) appear to differ from the results of Giovanelli, Haynes, Da Costa and collaborators (see the contribution by Da Costa to this volume). Understanding the sources of the differences is clearly an important goal.

2.5 Galaxy Pairwise Velocities

Using the cosmic virial theorem, the relative velocity dispersion of galaxy pairs can be used to estimate the matter density (*e.g.* Davis & Peebles 1983). The Las Campanas Redshift Survey (Shectman *et al.* 1996) contains about 26,000 redshifts out to $\sim 30,000$ km/sec and provides an excellent sample of galaxies not dominated by clusters. Davis (this conference) presented results based on this sample, concluding that relative galaxy pairs have a one-dimensional velocity dispersion of only 260 km/sec, implying $\Omega_m \sim 0.25$.

This method is very clean and conceptually simple; however, it again is limited by the assumption that bias is independent of scale. Moreover, Frenk (1997) argues that bulk velocity flows are not sensitive to Ω_m , and that the peculiar velocities are quite similar for a number of models with a range of values of Ω_m .

3 Ω_Λ and Ω_m Limits

The subject of the cosmological constant Λ has had a long and checkered history in cosmology. The reasons for skepticism regarding a non-zero value of the cosmological constant are many. First, there is a discrepancy of ≥ 120 orders of magnitude between current observational limits and estimates of the vacuum energy density based on current standard particle theory (*e.g.* Carroll, Press and Turner 1992). Second, it would require that we are now living at a special epoch when the cosmological constant has begun to affect the dynamics of the Universe (other than during a time of inflation). In addition, it is difficult to ignore the fact that historically a non-zero Λ has been dragged out prematurely many times to explain a number of other apparent crises, and moreover, adding

additional free parameters to a problem always makes it easier to fit data. Certainly the oft-repeated quote from Einstein to Gamov about his “biggest blunder” continues to undermine the credibility of a non-zero value for Λ .

However, despite the strong arguments that can be made for $\Lambda = 0$, there are compelling reasons to keep an open mind on the issue. First, at present there is no known physical principle that demands $\Lambda = 0$. Although supersymmetry can provide a mechanism, it is known that supersymmetry is broken (*e.g.*, Weinberg 1989). Second, unlike the case of Einstein’s original arbitrary constant term, standard particle theory and inflation now provide a physical interpretation of Λ : it is the energy density of the vacuum (*e.g.*, Weinberg 1989). Third, *if* theory demands $\Omega_{total} = 1$, then a number of observational results can be explained with a low Ω_m and $\Omega_m + \Omega_\Lambda = 1$: a) for instance, the observed large scale distribution of galaxies, clusters, large voids, and walls is in conflict with that predicted by the (standard) cold dark matter model for the origin of structure (*e.g.* Davis *et al.* 1992; Peacock & Dodds 1994); and b) the low values of the matter density based on a number of methods as described in §2. In addition, the discrepancy between the ages of the oldest stars and the expansion age can be resolved. Perhaps the most important reason to keep an open mind is that this is an issue that ultimately must be resolved by experiment.

The importance of empirically establishing whether there is a non-zero value of Λ cannot be overemphasized. However, it underscores the need for high-accuracy experiments: aspects of the standard model of particle theory have been tested in the laboratory to precisions unheard of in most measurements in observational cosmology. Nevertheless, cosmology offers an opportunity to test the standard model over larger scales and higher energies than can ever be achieved by other means. It scarcely needs to be said that overthrowing the Standard Model (*i.e.*, claiming a measurement of a non-zero value for Λ) will require considerably higher accuracy than is currently available.

What are the current observational limits on Ω_Λ ? In the next sections, limits based on both the observed numbers of quasars multiply imaged by galaxy “lenses” and limits from a sample of strongly lensed galaxies are briefly discussed.

3.1 Gravitational Lens Statistics

Fukugita, Futamase & Kasai (1990) and Turner (1990) suggested that a statistical study of the number density of gravitational lenses could provide a powerful test of a non-zero Λ . Subsequently a number of studies have been undertaken (*e.g.* Fukugita & Turner 1991; Bahcall *et al.* 1992; Maoz *et al.*

1993; Kochanek 1993, 1996). The basic idea behind this method is simple: the number of gravitationally lensed objects is a very sensitive function of Ω_Λ . For larger values of Ω_Λ , there is a greater probability that a quasar will be lensed because the volume over a given redshift interval is increased. In a flat universe with a value of $\Omega_\Lambda = 1$, approximately an order of magnitude more gravitational lenses are predicted than in a universe with $\Omega_\Lambda = 0$ (Turner 1990). Thus, simply counting the numbers of gravitationally lensed quasars can provide a very powerful limit on the value of Ω_Λ . In practice, however, there are a number of complications: galaxies evolve (and perhaps merge) with time, even elliptical galaxies contain dust, the properties of the lensing galaxies are not well-known (in particular, the dark matter velocity dispersion is unknown), and the numbers of lensing systems known at present is very small (~ 20). Moreover, while the predicted effects are very large for $\Omega_\Lambda = 1$, because the numbers are such a sensitive function of Ω_Λ , it is very difficult to provide limits below a value of about 0.6, given these complicating effects.

Kochanek (1996) has recently discussed these various effects in some detail, and investigated the sensitivity of the results to different lens models and extinction. His best estimated limits to date are : $\Omega_\Lambda < 0.66$ (95% confidence) for $\Omega_m + \Omega_\Lambda = 1$, and $\Omega_m = 0.15$ (90% confidence) if $\Omega_\Lambda = 0$. Significant improvements to these limits could be made by increasing the size of the current lens samples.

3.2 Strong Gravitational Lenses

A number of strong (elliptical galaxy) gravitational lens systems are known that may offer the potential of constraining the value of Ω_m and Ω_Λ through modeling of the lens properties. This method is less sensitive to Ω_Λ than the statistics of lensing, and again it is sensitive to a number of possible systematic effects: possible perturbations by cluster potentials, uncertainties in the underlying properties of the lensing galaxies, and model-dependent corrections due to evolution. The objects are faint and the errors in the luminosities and velocity dispersions are potentially very significant. A recent analysis of 7 strong lenses has been undertaken by Im *et al.* (1996). Their current results yield $\Omega_\Lambda = 0.64^{+0.15}_{-0.26}$ (*i.e.*, this measurement sits almost at the end of the range *excluded* by Kochanek (1996) at 95% confidence. Im *et al.* exclude $\Omega_m = 1.0$ at 97% confidence.

3.3 Ω_m and Ω_Λ from Type Ia Supernovae

The use of type Ia supernovae for measuring cosmological parameters is covered elsewhere in this volume by Filippenko (nearby supernovae and determinations

of H_0) and by Perlmutter (distant supernovae and Ω_m and Ω_Λ). Hence, these objects will not be discussed in much detail here, except to highlight their potential, and to summarize some of the main difficulties associated with them so that they can be compared relative to some of the other methods discussed in this review.

The obvious advantage of type Ia supernovae is the small dispersion in the Hubble diagram, particularly after accounting for differences in the overall shapes or slopes of the light curves (Phillips 1993; Hamuy *et al.* 1995; Reiss, Press & Kirshner 1997). In principle, separation of the effects of deceleration or a potential non-zero cosmological constant is straightforward, provided that (eventually) supernovae at redshifts of order unity can be measured with sufficient signal-to-noise and resolution against the background of the parent galaxies. The differences in the observed effects of Ω_m and Ω_Λ become increasingly easier to measure at redshifts exceeding ~ 0.5 . In principle, the evolution of single stars should be simpler than that of entire galaxies (that have been used for such measurements in the past).

At the present time, however, it is difficult to place any quantitative limits on the expected evolutionary effects for type Ia supernovae since the progenitors for these objects have not yet been unequivocally identified. Moreover, there may be potential differences in the chemical compositions of supernovae observed now and those observed at earlier epochs. In principle, such differences could be tested for empirically (as is being done for Cepheid variables, for example). It is also necessary to correct for obscuration due to dust (although in general, at least in the halos of galaxies, these effects are likely to be small; a minor worry might be that the properties of the dust could evolve over time). In detail, establishing accurate K-corrections for high-redshift supernovae, measuring reddenings, and correcting for potential evolutionary effects will be challenging, although, with the exception of measurements of the cosmic microwave background anisotropies (discussed in §9 below), type Ia supernovae may offer the best potential for measuring Ω_m and Ω_Λ .

The most recent results based on type Ia supernovae (Perlmutter *et al.* 1997) are encouraging, and they demonstrate that rapid progress is likely to be made in the near future. Currently, the published sample size is limited to 7 objects; however, many more objects have now been discovered. The feasibility of discovering these high-redshift supernovae with high efficiency has unquestionably been demonstrated (*e.g.* Perlmutter, this volume). However, systematic errors are likely to be a significant component of the error budget in the early stages of this program.

4 Summary of Current Ω_m and Ω_Λ Measurements

The results of the preceding sections on Ω_m and Ω_Λ are summarized graphically in Figure 2. The diagonal dashed line denotes a flat ($\Omega_m + \Omega_\Lambda = 1$) Universe. Plotted are the results from dynamical measurements (rotation curves, Local Group dynamics, galaxy velocity dispersions, X-ray clusters) that tend to give low values of $\Omega \sim 0.2$ - 0.3 . In addition, the preliminary results from the Perlmutter *et al.* (1997) type Ia supernova search are plotted with quoted 1σ error bars, along with the 95% limits ($\Omega_\Lambda < 0.66$) on Ω_m and Ω_Λ from gravitational lens statistics from Kochanek (1996), shown as an arrow along the diagonal.

What can be concluded about the value of Ω ? Given the available evidence and the remaining uncertainties, plus underlying assumptions at the present time, in my own view the data are still consistent with both an open and a flat Universe. This undesirable situation is very likely to be resolved in the near future with more accurate mapping of the anisotropies in the cosmic microwave background radiation (see §9). At this point in time, however, I believe that it is premature either to sound the death knell for (“standard”) inflationary theories or to conclude contrarily that an open Universe is not a viable option.

5 H_0 – The Hubble Constant

Sandage (1995) likens the measurement of H_0 to a game of chess. In chess, only a grand master “experiences a compelling sense of the issue and the best move. This player “knows” by intuition which clues are relevant... In other words his or her intuition judges what is real in the game, what will or will not lead to contradiction, and what aspects of the data to ignore.”

Although there are perhaps differences in philosophy and many different techniques for measuring H_0 , its importance cannot be underestimated. Knowledge of H_0 is required to constrain the estimates of the baryon density from nucleosynthesis at early epochs in the Universe. The larger the value of H_0 , the larger the component of non-baryonic dark matter is required, especially if the Universe has a critical density. The Hubble constant specifies both the time and length scales at the epoch of equality of the energy densities of matter and radiation. Both the scale at the horizon and the matter density determine the peak in the perturbation spectrum of the early universe. Hence, an accurate knowledge of the Hubble constant can provide powerful constraints on theories of the large-scale structure of galaxies. At present, large values of H_0 are problematic for the currently most successful models, those dominated by cold dark matter.

A value of H_0 to $\pm 1\%$ accuracy is still a goal far beyond currently avail-

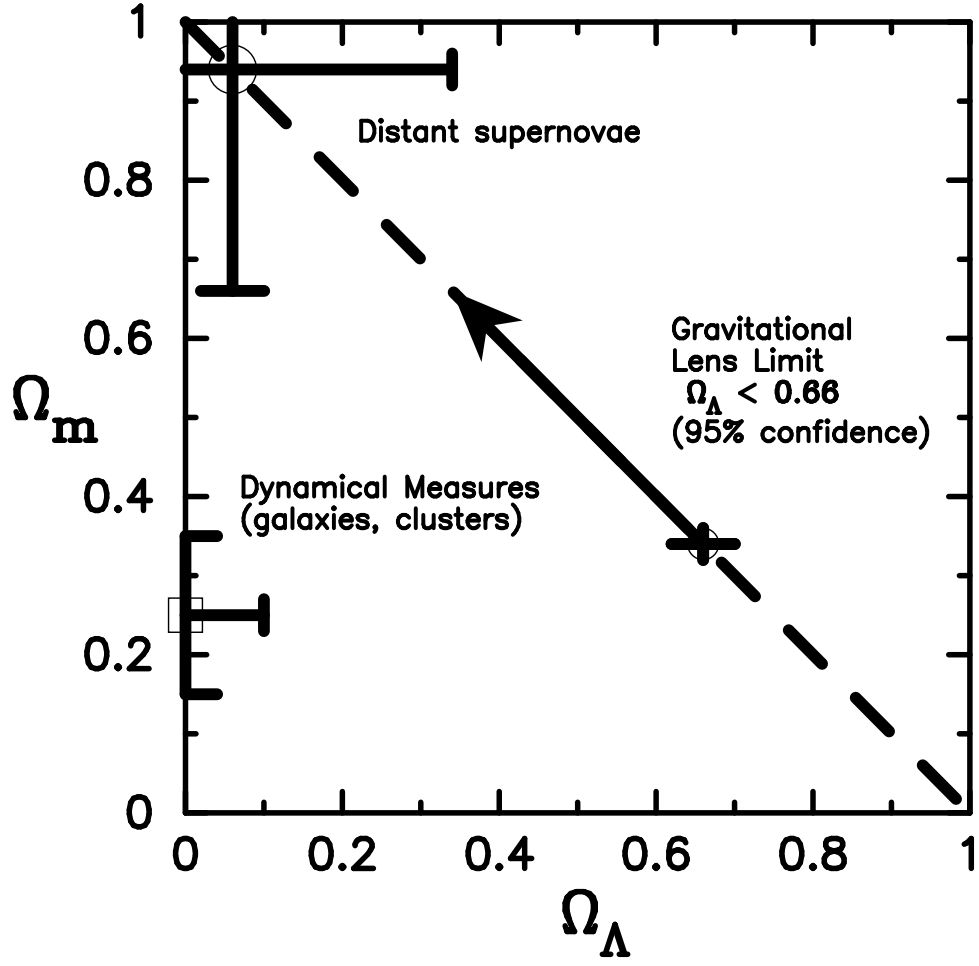


Figure 2: Summary of Omega Determinations. The dashed line corresponds to the case for a flat Universe: ($\Omega_m + \Omega_\Lambda = 1$). See text for details.

able measurement techniques. However, if, for example a value of $H_0 = 70$ km/sec/Mpc were confirmed at $\pm 1\%$ (95% confidence), *and* the ages of the oldest objects in the Universe were confirmed to be >12 Gyr, then a number of issues would be brought into tight focus (and corresponding new problems raised!). A cosmological constant would be *required*, there would be no further debate over the need for non-baryonic dark matter, and at least the standard version of cold dark matter would be ruled out (conclusively).

The requirements for measuring an accurate value of H_0 are simple to list in principle, but extremely difficult to meet in practice. As discussed in more detail in Freedman (1997), in general, there are 4 criteria that need to be met for any method. First, the method should be based upon well-understood physics; second, it should operate well into the smooth Hubble flow (velocity-distances greater than 10,000, and preferably, 20,000 km/sec); third, the method should be based on a statistically significant sample of objects, empirically established to have high internal accuracy; and finally, the method needs to be demonstrated empirically to be free of systematic errors. This list of criteria applies both to classical distance indicators as well as to other physical methods (in the latter case, for example, the Sunyaev Zel'dovich effect or gravitational lenses). The last point requires that several distance indicators meeting the first three criteria be available, but the current reality is that, unfortunately, at the present time, an ideal distance indicator or other method meeting all of the above criteria does not exist. The measurement of H_0 to $\pm 1\%$ is not yet possible; however, recent progress (reviewed below) illustrates that a measurement to $\pm 10\%$ is now feasible.

5.1 “Physical” versus “Astronomical” Methods

There is a common (mis)perception that some methods for determining H_0 based on simple physical principles are free from the types of systematics that often affect distance indicators (“physical” versus “astronomical” methods). However, the fact remains that aside from nearby geometric parallax measurements ($d < 100$ pc), *astrophysics enters all distance and H_0 determinations!* These methods include the gravitational lens time delay method, the Sunyaev Zel'dovich methods for clusters of galaxies, and theoretical modeling of type Ia and II supernovae.

For example, it is certainly true that the gravitational lensing method is premised on very solid physical principles (*e.g.* Refsdal 1964, 1966; Blandford & Narayan 1992). Unfortunately, the astronomical lenses are not idealized systems with well-defined properties that can be measured in a laboratory; they are galaxies whose underlying (luminous or dark) mass distributions are

not independently known, and furthermore they may be sitting in more complicated group or cluster potentials. A degeneracy exists between the mass distribution of the lens and the value of H_0 (*e.g.*, Kundić *et al.* 1997; Keeton and Kochanek 1997; Schechter *et al.* 1997). This is not a method based solely on well-known physics; it is a method that also requires knowledge of astrophysics. Ideally velocity dispersion measurements as a function of position are needed (to constrain the mass distribution of the lens). Such measurements are very difficult (and generally have not been available). Perhaps worse yet, the distribution of the dark matter in these systems is unknown. In a similar way, the Sunyaev-Zel’dovich method is sensitive to the clumping of X-ray gas, discrete radio sources, the projection of the clusters, and other astrophysical complications.

Hence the methods for measuring H_0 cannot be cleanly separated into purely “physical” and “astronomical” techniques. Rather, each method has its own set of advantages and disadvantages. In my view, it is vital to measure H_0 using a variety of different methods in order to identify potential systematic errors in any one technique. All methods require large, statistically significant samples. This is one of the current weakest aspects of the Sunyaev-Zel’dovich and gravitational-lens methods, for example, where samples of a few or only 2 objects, respectively, are currently available. In contrast, it is a clear disadvantage that many of the classical distance indicators (*e.g.*, the Tully-Fisher relation and at present, even the type Ia supernovae) do not have a well-understood physical basis. However, there are many cross-checks and tests for potential systematic effects that are now feasible and are being carried out for large samples of measured extragalactic distances (see §5.4 below). Assuming that systematic effects can eventually be understood and minimized, ultimately, the measurement of H_0 by a geometrical (or optical) technique at large distances will be crucial for establishing the reliability of the classical distance scale. For gravitational lenses, however, a considerable amount of work will be required to increase the numbers of systems with measured time delays, obtain velocity dispersion profiles for the faint lensing galaxies, constrain the lens models and test for other systematic effects, if this goal is to be reached.

Below, progress on H_0 measurements based on gravitational lenses, the Sunyaev Zel’dovich effect, and the extragalactic distance scale is briefly summarized.

5.2 Gravitational Lenses

Refsdæl (1964, 1966) noted that the arrival times for the light from two gravitationally lensed images of a background point source are dependent on the

path lengths and the gravitational potential traversed in each case. Hence, a measurement of the time delay and the angular separation for different images of a variable quasar can be used to provide a measurement of H_0 . This method offers tremendous potential because it can be applied at great distances and it is based on very solid physical principles. Moreover, the method is not very sensitive to Ω_m and Ω_Λ . Some of the practical difficulties in applying this method have already been discussed in the previous section.

A number of new results based on this technique have recently appeared. Estimates of time delay measurements are now available for 2 systems: 0957+561 (Kundić *et al.* 1997), and most recently, a new time delay has been measured for PG 1115 (Schechter *et al.* 1997; Keeton and Kochanek 1997).

In the case of 0957+561, progress has been made on several fronts. The time delay for this system has been a matter of some debate in the literature, with two different values of 410 and 536 days being advocated; extensive new optical data have now resolved this issue in favor of the smaller time delay ($\Delta t = 417 \pm 3$ days (Kundić *et al.* 1997). Another large observational uncertainty has been due to the difficulty of measuring an accurate velocity dispersion for the lensing galaxy. Recent data from the Keck telescope have provided a new measurement of the velocity dispersion (Falco *et al.* 1997). In addition, there has been substantial progress in modeling this system (Grogin & Narayan 1996). Based on the new time delay and velocity dispersions measurements, and the model of Grogin and Narayan, Falco *et al.* have recently derived a value of $H_0 =$ in the range $62 - 67 \pm 8$ km/sec/Mpc for this system. The velocity dispersion in the lensing galaxy appears to decrease very steeply as a function of position from the center of the galaxy; further higher-resolution measurements will be required to determine the reliability of these faint measurements.

Schechter *et al.* 1997 have undertaken an extensive optical monitoring program to measure two independent time delays in the quadruply-imaged quasar PG 1115+080. They fit a variety of models to this system, preferring a solution that yields a value of $H_0 = 42$ km/sec/Mpc $\pm 14\%$ (for $\Omega = 1$). The model in this case consists of fitting isothermal spheres to both the lensing galaxy and a nearby group of galaxies. They also considered additional models that yield values of $H_0 = 64$ and 84 km/sec/Mpc. Keeton & Kochanek (1997) have considered a wider class of models. They stress the degeneracies that are inherent in these analyses; a number of models with differing radial profiles for the lensing galaxy and group, and with differing positions for the group, yield fits with chi-squared per degrees of freedom less than 1. They conclude that $H_0 = 60 \pm 17$ km/sec/Mpc ($1-\sigma$).

5.3 *Sunyaev Zel'dovich Effect and X-Ray Measurements*

The inverse-Compton scattering of photons from the cosmic microwave background off of hot electrons in the X-ray gas of rich clusters results in a measurable decrement in the microwave background spectrum known as the Sunyaev-Zel'dovich (SZ) effect (Zel'dovich and Sunyaev 1969). Given a spatial (preferably 2-dimensional) distribution of the SZ effect and a high-resolution X-ray map, the density and temperature distributions of the hot gas can be obtained; the mean electron temperature can be obtained from an X-ray spectrum. An estimate of H_0 can be made based on the definitions of the angular-diameter and luminosity distances. The method makes use of the fact that the X-ray flux is distance-dependent, whereas the Sunyaev-Zel'dovich decrement in the temperature is not.

Once again, the advantages of this method are that it can be applied at large distances and, in principle, it has a straightforward physical basis. As discussed in §5.1, some of the main uncertainties with this method are due to potential clumpiness of the gas (which would result in reducing H_0), projection effects (if the clusters observed are prolate, H_0 could be larger), the assumption of hydrostatic equilibrium, details of the models for the gas and electron densities, and potential contamination from point sources.

To date, a range of values of H_0 have been published based on this method ranging from $\sim 25 - 80$ km/sec/Mpc (*e.g.*, McHardy *et al.* 1990; Birkinshaw & Hughes 1994; Rephaeli 1995; Herbig, Lawrence & Readhead 1995). The uncertainties are still large, but as more and more clusters are observed, higher-resolution (2D) maps of the decrement, and X-ray maps and spectra become available, the prospects for this method will continue to improve. At this conference, Carlstrom reported on a new extensive survey of lenses being undertaken both at Hat Creek and the Owens Valley Radio Observatory. X-ray images are being obtained with ROSAT and X-ray spectra with ASCA.

5.4 *The Cepheid-Calibrated Extragalactic Distance Scale*

Establishing accurate extragalactic distances has provided an immense challenge to astronomers since the 1920's. The situation has improved dramatically as better (linear) detectors have become available, and as several new, promising techniques have been developed. For the first time in the history of this difficult field, relative distances to galaxies are being compared on a case-by-case basis, and their quantitative agreement is being established. Several, detailed reviews on this progress have been written (see, for example, the conference proceedings for the Space Telescope Science Institute meeting on the Extragalactic Distance Scale edited by Donahue and Livio 1997).

The Hubble Space Telescope (HST) Key Project on H_0 has been designed to undertake the calibration of a number of secondary distance methods using Cepheid variables (Freedman *et al.* 1994; Kennicutt, Freedman & Mould 1995; Mould *et al.* 1995). Briefly, there are three primary goals: (1) To discover Cepheids, and thereby measure accurate distances to spiral galaxies suitable for the calibration of several independent secondary methods. (2) To make direct Cepheid measurements of distances to three spiral galaxies in each of the Virgo and Fornax clusters. (3) To provide a check on potential systematic errors both in the Cepheid distance scale and the secondary methods. The final goal is to derive a value for the the Hubble constant, to an accuracy of 10%. Cepheids are also being employed in several other HST distance scale programs (*e.g.*, Sandage *et al.* 1996; Saha *et al.* 1994, 1995, 1996; and Tanvir *et al.* 1995).

In Freedman, Madore & Kennicutt (1997), a comparison of Cepheid distances is made with a number of other methods including surface-brightness fluctuations, the planetary nebula luminosity function, tip of the red giant branch, and type II supernovae. (Extensive recent reviews of all of these methods can be found in Livio and Donahue (1997); by Tonry; Jacoby; Madore, Freedman & Sakai; Kirshner). In general, there is excellent agreement amongst these methods; the relative distances agree to within $\pm 10\%$ (1-sigma). The use of both type Ia and type II supernovae for the purposes of determining H_0 are described in this volume by Filippenko.

The results of the H_0 Key Project have been summarized recently by Freedman, Madore & Kennicutt (1997); Mould *et al.* (1997); and Freedman (1997). For somewhat different views, see Sandage & Tammann (1997). The remarks in the rest of this section follow Freedman (1997). At this mid-term point in the HST Key Project, our results yield a value of $H_0 = 73 \pm 6$ (statistical) ± 8 (systematic) km/sec/Mpc. This result is based on a variety of methods, including a Cepheid calibration of the Tully-Fisher relation, type Ia supernovae, a calibration of distant clusters tied to Fornax, and direct Cepheid distances out to ~ 20 Mpc. In Table 2 the values of H_0 based on these various methods are summarized.

These recent results on the extragalactic distance scale are very encouraging. A large number of independent secondary methods (including the most recent type Ia supernova calibration by Sandage *et al.* 1996) appear to be converging on a value of H_0 in the range of 60 to 80 km/sec/Mpc. The long-standing factor-of-two discrepancy in H_0 appears to be behind us. However, these results underscore the importance of reducing remaining errors in the Cepheid distances (*e.g.*, those due to reddening and metallicity corrections),

Table 2: SUMMARY OF KEY PROJECT RESULTS ON H_0

Method	H_0
Virgo	80 ± 17
Coma via Virgo	77 ± 16
Fornax	72 ± 18
Local	75 ± 8
JT clusters	72 ± 8
SN Ia	67 ± 8
TF	73 ± 7
SN II	73 ± 7
$D_N - \sigma$	73 ± 6
Mean	73 ± 4
Systematic Errors	± 4 ± 4 ± 5 ± 2 (LMC) ([Fe/H]) (global) (photometric)

Table 3: Current values of H_0 for various methods. For each method, the formal statistical uncertainties are given. The systematic errors (common to all of these Cepheid-based calibrations) are listed at the end of the table. The dominant uncertainties are in the distance to the LMC and the potential effect of metallicity on the Cepheid period-luminosity relations, plus an allowance is made for the possibility that the locally measured value of H_0 may differ from the global value. Also allowance is made for a systematic scale error in the photometry which might be affecting all software packages now commonly in use. Our best current weighted mean value is $H_0 = 73 \pm 6$ (statistical) ± 8 (systematic) km/sec/Mpc.

since at present the majority of distance estimators are tied in zero point to the Cepheid distance scale. A $1\text{-}\sigma$ error of $\pm 10\%$ on H_0 (the aim of the Key Project) currently amounts to approximately $\pm 7 \text{ km/sec/Mpc}$, and translates into a 95% confidence interval on H_0 of roughly 55 to 85 km/sec/Mpc.

While this is an enormous improvement over the factor-of-two disagreement of the previous decades, it is not sufficiently precise, for example, to discriminate between current models of large scale structure formation, to resolve definitively the fundamental age problem, or to settle the question of a non-zero value of Λ . Before compelling constraints can be made on cosmological models, it is imperative to rule out remaining sources of systematic error in order to severely limit the alternative interpretations that can be made of the data. The spectacular success of HST, and the fact that a value of H_0 accurate to 10% ($1\text{-}\sigma$) now appears quite feasible, also brings into sharper focus smaller (10-15%) effects which were buried in the noise during the era of factor-of-two discrepancies. Fortunately, a significant improvement will be possible with the new infrared capability afforded by the recently augmented near-infrared capabilities of HST (the NICMOS instrument). Planned NICMOS observations will reduce the remaining uncertainties due to both reddening and metallicity by a factor of 3.

6 t_0 - Ages of the Oldest Stars

The ages of stars can be derived quite independently from the expansion age of the Universe (obtained by integrating the Friedmann equation), and have long been used as a point of comparison and constraint on cosmology; for example, globular cluster age-dating, nucleocosmochronology, and white-dwarf cooling estimates for the Galactic disk. The reader is referred to earlier reviews on these topics by Renzini (1991), Schramm (1989). For the purposes of this review, I briefly consider only two types of age determinations: those based on Galactic globular clusters, and a new estimate of the age based on a measurement of radioactive thorium in a metal poor Galactic halo star.

6.1 Globular Cluster Ages

There are also many excellent recent reviews covering in great detail the ages obtained for Galactic globular clusters (*i.e.*, from a comparison of observed color magnitude diagrams and theoretical evolution models). At the moment, there is a fairly broad consensus that Galactic globular clusters are most likely at least 14-15 Gyr old (*e.g.* Chaboyer *et al.* 1996; VandenBerg *et al.* 1996; Shi 1995).

It is not widely appreciated that *the largest uncertainty in the globular-cluster ages results from uncertainties in the distances to the globular clusters*, which currently are based on statistical parallax measurements of Galactic RR Lyrae stars or on parallaxes for nearby subdwarfs (*e.g.* Renzini, 1991; Chaboyer *et al.* 1996; VandenBerg *et al.* 1996). Although the ages of globular clusters are widely regarded as theoretically-determined quantities, in the process of determining ages, it is still necessary to interface theory with observation and transform the observed globular cluster magnitudes to bolometric luminosities (via an accurate distance scale). The subdwarf and RR Lyrae statistical parallax distance calibrations currently differ by about ~ 0.25 - 0.30 mag. Unfortunately, as emphasized by Renzini, small errors in distance modulus (0.25 mag or 13% in distance) correspond to 25% differences in age. Even with improved parallax measurements (for example, soon to be available from HIPPARCHOS), there are many subtle issues (*e.g.*, reddening, metallicity, photometric zeropoints) that combine to make it a very difficult problem to achieve distances to better than 5% accuracy.

As discussed previously in many contexts (*e.g.* Walker 1992; Freedman & Madore 1993; van den Bergh 1995, and most recently by Feast & Catchpole 1997), there is also currently a discrepancy in the Cepheid and RR Lyrae distances to nearby galaxies. If the Cepheid distances are correct, it would imply that the absolute magnitudes of RR Lyraes are brighter (by about 0.3 mag) than suggested by statistical parallax and Baade-Wesselink calibrations for Galactic RR Lyraes (*e.g.* see VandenBerg, Bolte & Stetson 1996 for a recent discussion). This brighter RR Lyrae calibration agrees well in zero point with that from Galactic subdwarfs. Based on the models of VandenBerg *et al.* 1997, applying this calibration (adopting $M_V(\text{RR})=0.40$ mag) to the metal-poor globular cluster M92, results in an age of 15.8 ± 2 Gyr. If the fainter RR Lyrae distance scale is correct, the age derived for M92 based on these same recent models increases to ~ 19 Gyr. Alternatively, if the Feast & Catchpole calibration of Galactic Cepheids based on HIPPARCHOS parallaxes is correct, then the resulting RR Lyrae calibration is even brighter ($M_V(\text{RR})=0.25$ at $[\text{Fe}/\text{H}] = -1.9$), and the corresponding age for M92 would be reduced to about 13 Gyr (based on the same Vandenberg models). A new calibration of Galactic metal-poor subdwarfs, also based on new HIPPARCHOS parallaxes, appears to confirm these younger ages (Reid, private communication). It is interesting to note that while the distances to nearby galaxies have converged to a level where they no longer have a factor-of-two impact on the Hubble constant, subtle differences of only a few tenths of a magnitude in distance modulus can still have very significant impact on cosmology, through the ages determined from stellar evolution.

6.2 Thorium Ages

A new measurement of the age of a very metal poor star in the halo of our Galaxy has recently been made by Cowan *et al.* (1997), following a technique introduced by Butcher (1987). These authors make use of very high-resolution echelle spectra of CS22892-052, a star with a metallicity of only $[\text{Fe}/\text{H}] = -3.1$. They find that the observed abundances for stable elements in this star match the observed r-process elemental abundances observed in the Sun. However, for the radioactive element thorium, the abundance is down by a factor of 40 relative to solar. Allowing for the radioactive decay of thorium relative to (stable) europium yields a minimum age for this star of 15.2 ± 3.7 Gyr (1-sigma). If instead of europium alone, an average abundance for all r-process elements from Eu-Er is used, an age of 13.8 ± 3.7 Gyr results. This lower limit to the age is independent of any model of Galactic evolution (which only serve to increase the total age estimates for the Universe). It depends on both the decay rate and the initial abundance of thorium. Although the current sample is small (1 star!) and the uncertainties are correspondingly large, there is excellent promise for the future once the sample is enlarged. Methods like this one are particularly important because of the opportunity of having high-quality ages completely independent of the globular cluster age scale.

7 Remaining Issues for Measuring t_0

What are the ages of the oldest objects in the Universe? In this context, we need to keep in mind that it is currently only a useful working hypothesis that the Galactic globular clusters are representative of the oldest objects in the Universe (*e.g.* see Freedman 1995 for a more detailed discussion). Currently, the sample of objects for which direct (*i.e.* main-sequence-fitting) ages can be measured is limited to our own Galaxy and a small number of satellites around our own Galaxy. It is at least conceivable that in denser environments in the early Universe, star formation could have proceeded earlier than for Galactic globular clusters. At this time, there is no direct information with which to constrain the true dispersion in (or upper limit to) ages in environments outside the nearest galaxies in our own Local Group. There are, for example, no giant elliptical galaxies in the Local Group. Although considerable effort is now being invested in finding potential ways to lower the Galactic globular cluster ages, there is reason to keep in mind that the expansion-age discrepancy could potentially be even worse than is currently being discussed.

8 $H_0 t_0$

One of the most powerful tests for a non-zero cosmological constant is provided by a comparison of the expansion and oldest-star ages. To quote Carroll, Press and Turner (1990), “A high value of H_0 (>80 km/s/Mpc, say), combined with no loss of confidence in a value 12-14 Gyr as a *minimum* age for some globular clusters, would effectively prove the existence of a significant Ω_Λ term. Given such observational results, we know of no convincing alternative hypotheses.”

In Figure 3, the dimensionless product of $H_0 t_0$ is plotted as a function of Ω . Two different cases are illustrated: an open $\Omega_\Lambda = 0$ Universe, and a flat Universe with $\Omega_\Lambda + \Omega_m = 1$. Suppose that both H_0 and t_0 are both known to $\pm 10\%$ (1- σ , *including systematic errors*). The dashed and dot-dashed lines indicate 1- σ and 2- σ limits, respectively for values of $H_0 = 70$ km/sec/Mpc and $t_0 = 15$ Gyr. Since the two quantities H_0 and t_0 are completely independent, the two errors have been added in quadrature, yielding a total uncertainty on the product of $H_0 t_0$ of $\pm 14\%$ *rms*. These values of H_0 and t_0 are consistent with a Universe where $\Omega_\Lambda = 0.8$, $\Omega_m = 0.2$. The Einstein-de Sitter model ($\Omega_m = 1$, $\Omega_\Lambda = 0$) is excluded (at 2.5σ).

Despite the enormous progress recently in the measurements of H_0 and t_0 , Figure 3 demonstrates that significant further improvements are still needed. First, in the opinion of this author, *total* (including both statistical and systematic) uncertainties of $\pm 10\%$ have yet to be achieved for either H_0 or t_0 . Second, assuming that such accuracies will be forthcoming in the near future for H_0 (as the Key Project, supernova programs and other surveys near completion), and for t_0 (as HIPPARCHOS provides an improved calibration both for RR Lyraes and subdwarfs), it is clear from this figure that if H_0 is as high as 70 km/sec/Mpc, then accuracies of significantly *better than* $\pm 10\%$ will be required to rule in or out a non-zero value for Λ . (If H_0 were larger (or smaller), this discrimination would be simplified!)

9 Cosmological Parameters from Cosmic Microwave Background Anisotropies

One of the most exciting future developments with respect to the accurate measurement of cosmological parameters will be the opportunity to measure anisotropies in the cosmic microwave background to high precision. Planned balloon-born experiments (*e.g.*, MAX, MAXIMA, and Boomerang) will shortly measure the position of the first acoustic peak in the cosmic background anisotropy spectrum. Even more promising are future satellite experiments (*e.g.*, MAP to be launched by NASA in 2000, and the European COBRAS/SAMBA

H₀ and t₀ Measurements to ±10%

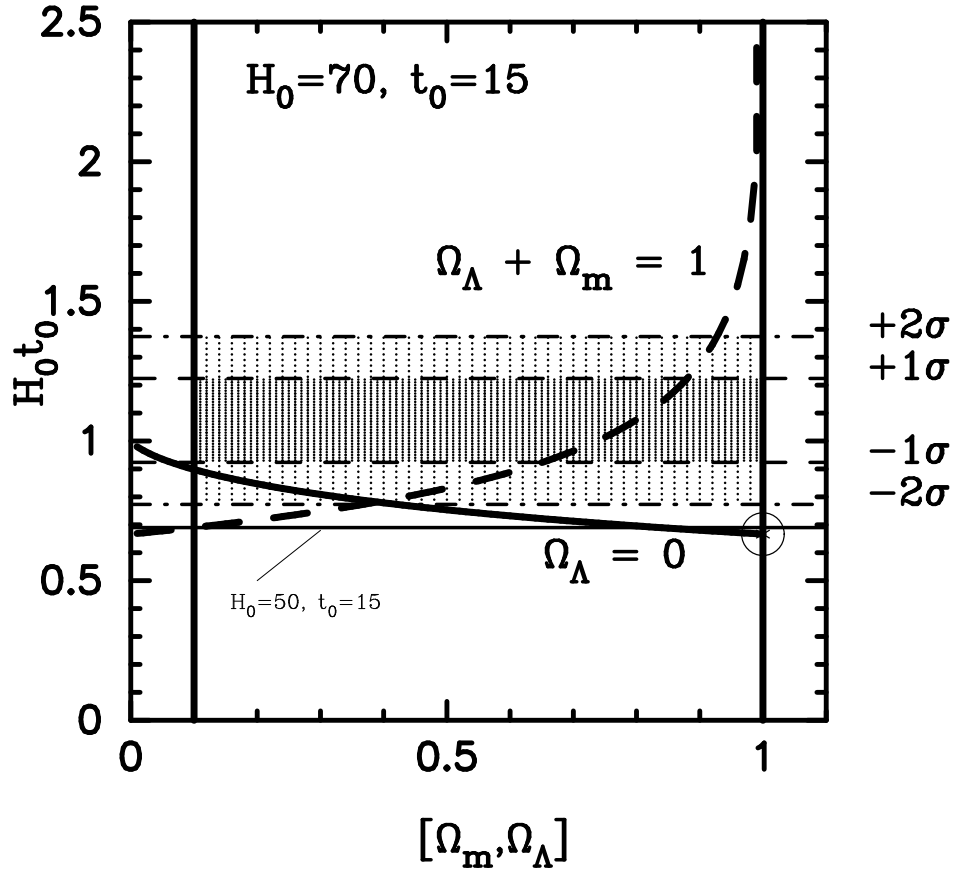


Figure 3: The product of $H_0 t_0$ as a function of Ω . The dashed curve indicates the case of a flat Universe with $\Omega_\Lambda + \Omega_m = 1$. The abscissa in this case corresponds to Ω_Λ . The solid curve represents a Universe with $\Omega_\Lambda = 0$. In this case, the abscissa should be read as Ω_m . The dashed and dot-dashed lines indicate $1\text{-}\sigma$ and $2\text{-}\sigma$ limits, respectively for values of $H_0 = 70$ km/sec/Mpc and $t_0 = 15$ Gyr in the case where both quantities are known to $\pm 10\%$ ($1\text{-}\sigma$). The large open circle denotes values of $H_0 t_0 = 2/3$ and $\Omega_m = 1$ (*i.e.*, those predicted by the standard Einstein-de Sitter model). Also shown for comparison is a solid line for the case $H_0 = 50$ km/sec/Mpc, $t_0 = 15$ Gyr.

mission, now renamed the PLANCK Surveyor mission, currently planned to be launched in 2005).

The underlying physics governing the shape of the anisotropy spectrum is that describing the interaction of a very tightly coupled fluid composed of electrons and photons before (re)combination (*e.g.*, Hu & White 1996; Sunyaev & Zel'dovich 1970). It is elegant, very simple in principle, and offers extraordinary promise for measuring cosmological parameters; (*e.g.*, H_0 , Ω_0 , and the baryon density Ω_b to precisions of 1% or better: Bond, Efstathiou & Tegmark 1997).

The final accuracies will of course (again) depend on how well various systematic errors can be controlled or eliminated. The major uncertainties will be determined by how well foreground sources can be subtracted, and probably to a lesser extent, by calibration and instrumental uncertainties. (PLANCK will provide a cross check of the MAP calibration.) Potentially the greatest problem is the fact that extracting cosmological parameters requires a specific model for the fluctuation spectrum. Currently the estimates of the precisions (*i.e.*, without systematic effects included) are based on models in which the primordial fluctuations are Gaussian and adiabatic, and for which there is no preferred scale. A very different anisotropy power spectrum shape is predicted for defect theories (Turok 1996), but these calculations are more difficult and have not yet reached the same level of predictive power. Important additional constraints will come from polarization measurements *e.g.*, Zaldarriaga, Spergel & Seljak 1997; Kamionkowski *et al.* 1997). The polarization data will provide a means of breaking some of the degeneracies amongst the cosmological parameters that are present in the temperature data alone. Furthermore, they are sensitive to the presence of a tensor (gravity wave) contribution, and hence will allow a very sensitive test of inflationary models.

Figure 4 shows a plot of the predicted angular power spectrum for cosmic microwave background (CMB) anisotropies reproduced from Hu, Sugiyama, & Silk (1997). The position of the first acoustic peak is very sensitive to the value of Ω_0 , and, as noted by these authors, the spacing between the acoustic peaks in the power spectrum appears to provide a fairly robust measure of Ω_0 . The accurate determination of other cosmological parameters will require the measurement of peaks at smaller (arcminute) angular scales. In general, the ratio of the first to the third peaks is sensitive to the value of H_0 (*e.g.*, Hu & White 1996). Excellent sky coverage is critical to these efforts in order to reduce the sampling variance.

Can the cosmological parameters be measured to precisions of $\leq 1\%$ with currently planned experiments as advertised above? I believe that both MAP and PLANCK are likely to revolutionize our understanding of cosmology. Ob-

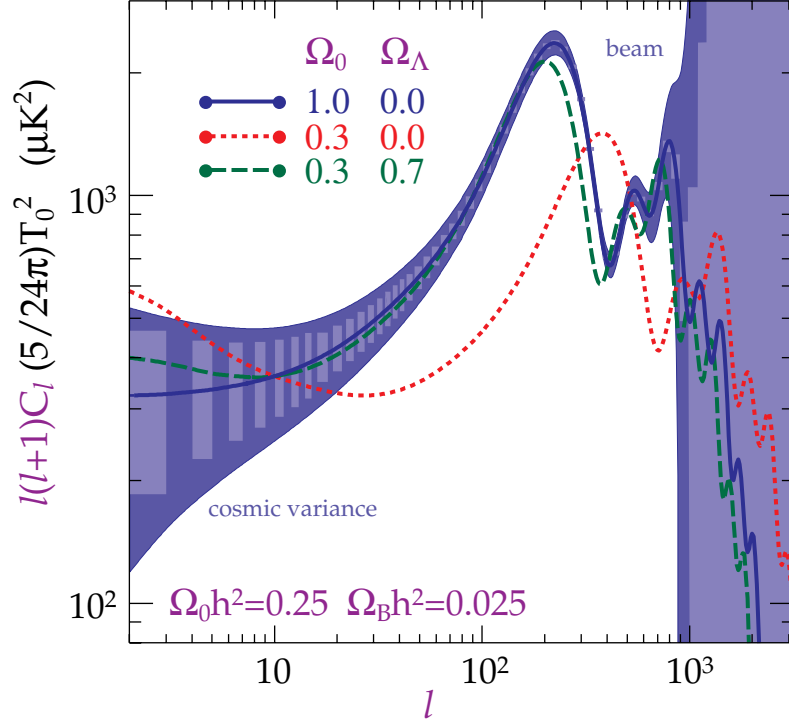


Figure 4: The angular power spectrum of cosmic microwave background anisotropies assuming adiabatic, nearly scale-invariant models for a range of values of Ω_0 and Ω_Λ (Hu, Sugiyama, and Silk 1997; their Figure 4). The C_l values correspond to the squares of the spherical harmonics coefficients. Low l values correspond to large angular scales ($l \sim \frac{200^\circ}{\theta}$). The position of the first acoustic peak is predicted to be at $l \sim 220\Omega_{TOT}^{-1/2}$, and hence, shifts to smaller angular scales for open universes.

servation of a Gaussian, adiabatic fluctuation spectrum would be a stunning confirmation of the “standard” cosmology. However, equally fundamental would be the case where the observed anisotropy spectrum resembles nothing like those for any of the various current theoretical predictions. In the former case, *if* foreground effects can be accounted for, then measurement of the cosmological parameters to these levels of precision will eventually follow. However, in the latter case, at least until the origin of the spectrum could be predicted from first principles, all bets would be off for the determination of cosmological parameters.

Can the foreground subtraction be accounted for accurately enough to yield final accuracies of 1% (or better)? There will be foreground contributions due to faint, diffuse Galactic emission. MAP will have 5 frequency bands ranging from 22 to 90 GHz allowing both the spectral and spatial distribution of the Galactic foreground to be measured. PLANCK will have 9 frequency channels from 30 GHz to 900 GHz. However, there are many sources of foregrounds whose subtraction is critical; perhaps the greatest unknown is the potential contribution from GHz radio sources, many of which could potentially also be variable sources. Deep 90 GHz radio surveys from the ground might address the question of how serious an issue such sources could be (Spergel, private communication). Although MAP will cover any given region of the sky several times, the signal-to-noise for an individual image will be insufficient to detect any but the brightest sources. In addition there will be foreground contributions due to diffuse emission from external galaxies, dust within galaxies, and bright infrared luminous galaxies. Until these experiments are completed, it will be difficult to assess whether these systematic uncertainties are likely to be small relative to the quoted formal uncertainties.

10 Summary

The current best measurements for the cosmological parameters yield:

$$\begin{array}{ll} \Omega_m \sim (0.2 - 0.4) \pm 0.1 & (1-\sigma) \\ H_0 \sim (67 - 73) \pm 7 \text{ km/sec/Mpc} & (1-\sigma) \\ t_0 \sim (14 - 15) \pm 2 \text{ Gyr} & (1-\sigma) \\ \Omega_\Lambda < 0.7 & (2-\sigma) \end{array}$$

The low value for Ω_m and relatively high value for $H_0 t_0$ do not favor the standard Einstein-de Sitter ($\Omega_m = 1, \Omega_\Lambda = 0$) Universe; however, this model cannot be ruled out at high statistical significance. Moreover, systematic errors are still a source of serious concern. If the new HIPPARCHOS calibrations are confirmed, the ages of globular clusters may be as low as 10-12 Gyr. Rapid progress is expected in addressing these systematic effects; in particular new data from

HST, HIPPARCHOS, and MAP/PLANCK offer the enticing possibility that all of the cosmological parameters may soon be measured to unprecedented accuracies of $\pm 1\text{-}5\%$ within a decade. Let us hope that unexpected systematic errors will not continue to lurk (as they have done historically so many times before) in these future efforts to define the basic cosmological parameters.

Acknowledgments

It is a pleasure to thank the organizing committee for an extremely enjoyable and interesting conference, and for the opportunity to speak. The work presented on the Cepheid-based extragalactic distance scale (§5.4) has been done in collaboration with the Hubble Space Telescope Key Project team on the Extragalactic Distance Scale and I would like to acknowledge the contributions of R. Kennicutt, J.R. Mould (co-PI's), S. Faber, L. Ferrarese, H. Ford, B. Gibson, J. Graham, J. Gunn, M. Han, J. Hoessel, J. Huchra, S. Hughes, G. Illingworth, B.F. Madore, R. Phelps, A. Saha, S. Sakai, N. Silberman, and P. Stetson, and graduate students F. Bresolin, P. Harding, D. Kelson, L. Macri, D. Rawson, and A. Turner. This work is based on observations with the NASA/ESA Hubble Space Telescope, obtained by the Space Telescope Science Institute, which is operated by AURA, Inc. under NASA contract No. 5-26555. Support for this work was provided by NASA through grant GO-2227-87A from STScI.

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